Response of Simple Spans to Moving Mass Loads

JAMES F. WILSON*
Duke University, Durham, N.C.

Theme

A SIMPLE, prestressed span is initially horizontal under self-weight. Distributed masses traverse the span at constant speed, and the span responses are sought. This system simulates the traverse of air cushion vehicles, most of whose weight is unsprung and concentrated just above an elastic guideway. Present results are based on Ref. 1 whereas related work is summarized in Refs. 2 and 3.

Contents

A mass M of length d travels at constant speed v over identical spans of length s, as shown in Fig. 1. Span motion is based on Bernoulli-Euler beam theory or

$$EI\frac{\partial^4 y}{\partial x^4} + T_c\frac{\partial^2 y}{\partial x^2} + Ky + c\frac{\partial y}{\partial t} + \rho A\frac{\partial^2 y}{\partial t^2} = f(x,t) \tag{1}$$

where (x, y) are the span coordinates, t is time, EI is the stiffness, T_c is the constant axial compressive load, K is the elastic foundation constant, c is the viscous damping constant, ρ is the span mass density, and A is the span's cross sectional area. If $\ddot{y}_{0m}$ is the absolute vertical acceleration at the midpoint of the mth mass segment located at distance z_m, then the loading $f(x,t)$ for the ith span is

$$f_i(x,t) = q_m(t)f_m(x) \tag{2a}$$

where

$$m = \alpha_1, \alpha_1+1, \ldots \alpha_2-1, \alpha_2$$

$$q_m(t) = (1/d)(Mg - M\ddot{y}_{0m}) \tag{2b}$$

$$f_m(x) = \begin{cases} 0 \text{ for } \left(x_{im}+\dfrac{d}{2}\right) < x \leqq \left(x_{im}-\dfrac{d}{2}\right) \\ 1 \text{ for } \left(x_{im}-\dfrac{d}{2}\right) < x \leqq \left(x_{im}+\dfrac{d}{2}\right) \end{cases} \tag{2c}$$

and

$$x_{im} = z_m - (i-1)s \tag{2d}$$

Here, α_2 is the number of the first mass system having any of its pressure on span i and α_1 is the last mass system having any of its pressure on span i. The boundary conditions are

$$y(0,t) = y(s,t) = \partial^2 y(0,t)/\partial x^2 = \partial^2 y(s,t)/\partial x^2 = 0 \tag{3}$$

for vanishing displacements and moments at each end, $x = 0, s$. It is assumed that each span is at rest before the mass front reaches it, so that the initial conditions at time $t = t_0$ are

$$y(x,t_0) = \partial y(x,t_0)/\partial t = 0 \tag{4}$$

For the ith span, where $y = y(x_{im}, t)$

$$\ddot{y}_{0m} = \partial^2 y/\partial t^2 + 2v(\partial^2 y/\partial x^2) + v^2\, \partial^2 y/\partial x^2 \tag{5}$$

Received March 31, 1972; presented as Paper 72-338 at the AIAA/ASME/SAE 13th Structures, Structural Dynamics, and Materials Conference, San Antonio, Texas, April 10–12, 1972; synoptic received May 10, 1972; revision received August 21, 1972. Full paper available from AIAA Library, 750 Third Avenue, New York, N.Y. 10017. Price: Microfiche, $1.00; hard copy, $5.00. **Order must be accompanied by remittance.** Sponsored by the U.S. Department of Transportation, contract DOT-FR-00037. Computer results due to S. B. Biggers and N. G. Awadalla.

Index category: Structural Dynamic Analysis.

* Associate Professor, Civil Engineering Department.

Closed form solutions for y in terms of $\ddot{y}_{0m}$ were obtained from Eq. (1) where conditions (3) and (4) were satisfied. Numerical solutions compatible with Eqs. (2) and (5) were obtained by an iterative process in which the whole system of traveling masses reached a steady state condition after traversing several consecutive spans. Moment responses $M(x,t)$ were calculated in the same way, where

$$M(x,t) = -EI\,\partial^2 y/\partial x^2 \tag{6}$$

Numerical results were obtained for elevated spans for which $K = 0$ and $T_c = 0$, where the undamped span frequency is

$$p_{00} = (\pi^4 EI/s^4 \rho A)^{1/2} \quad \text{(rad/time)} \tag{7}$$

and the passage frequency is

$$\omega_1 = \pi v/s \quad \text{(rad/time)} \tag{8}$$

Span responses are governed by the dimensionless parameters

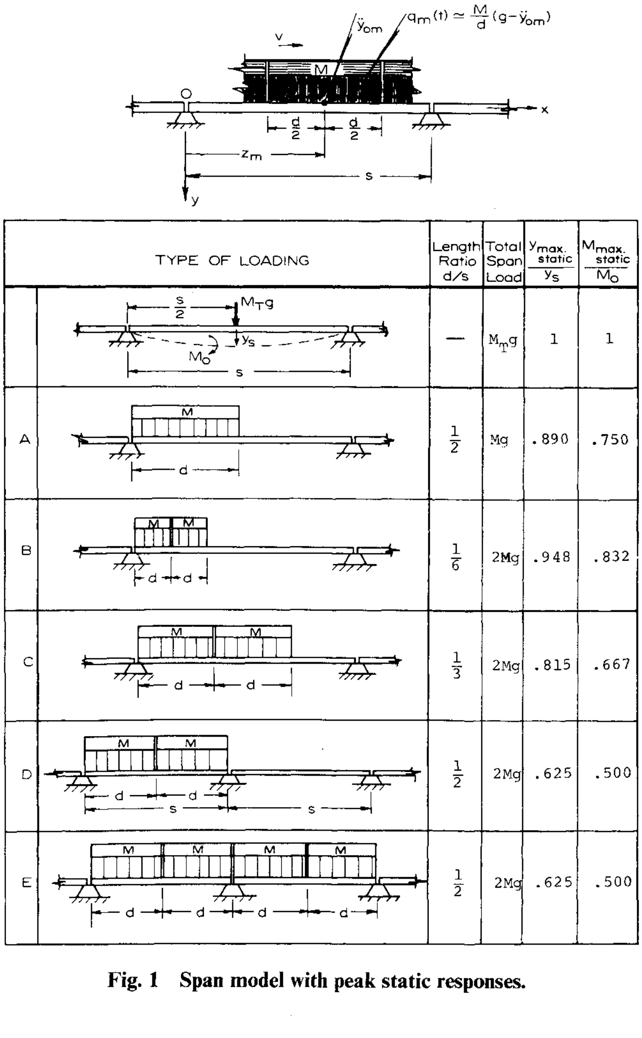
$$\omega_1/p_{00} \quad \text{Frequency ratio} \tag{9a}$$

$$M/\rho As \quad \text{Mass ratio} \tag{9b}$$

$$\xi_0 = (1/p_{00})\cdot(c/2\rho A) \quad \text{Damping ratio} \tag{9c}$$

$$s/d \quad \text{Length ratio} \tag{9d}$$

Responses for cases A through E of Fig. 1 are based on their



	Length Ratio d/s	Total Span Load	$y_{max.\ static}/y_s$	$M_{max.\ static}/M_o$
	—	$M_T g$	1	1
A	$\frac{1}{2}$	Mg	.890	.750
B	$\frac{1}{6}$	2Mg	.948	.832
C	$\frac{1}{3}$	2Mg	.815	.667
D	$\frac{1}{2}$	2Mg	.625	.500
E	$\frac{1}{2}$	2Mg	.625	.500

Fig. 1 Span model with peak static responses.

Table 1 Parametric study of span responses

Case (Fig. 1)	Mass ratio $M_T/\rho As$	Frequency ratio ω_1/p_{00}	Responses $y_{max}/y_{max\ static}$	$M_{max}/M_{max\ static}$
A	1/4	0.865	2.09	2.11
		1.22	1.78	2.00
		1.72	1.54	2.02
B	1/4	0.865	1.91	1.88
		1.22	1.71	1.95
		1.72	1.46	2.07
C	1/4	0.865	1.93	1.94
		1.22	1.88	2.02
		1.72	1.64	1.98
	1/2	0.865	2.45	2.15
		1.22	2.27	2.66
		1.22	2.09[a]	2.46[a]
		1.72	2.25	3.40
D	1/4	0.865	1.97	2.12
		1.22	2.16	2.38
		1.72	2.03	2.26
	1/2	0.865	2.35	2.67
		1.22	2.87	3.27
		1.22	2.63[a]	2.95[a]
		1.72	3.08	3.84
E	1/4	0.865	2.12	2.16
		1.22	2.90	3.00
		1.72	4.55	5.00
	1/2	0.865	2.74	2.97
		1.22	5.40	5.90
		1.22	4.90[a]	5.45[a]
		1.72	11.0	11.0

[a] $\xi_0 = 0.05$; otherwise, $\xi_0 = 0$.

respective maximum static responses, all referring to the point load values for deflection y_s and bending moment M_0 or

$$y_s = (M_T g \cdot s^3)/48EI, \qquad M_0 = (M_T g \cdot s)/4 \tag{10}$$

where M_T is total mass on a single span.

Typical response curves for case C are shown in Fig. 2 based on $M_T/\rho As = 1/2$ and $K = T_c = \xi_0 = 0$. The ordinates are the peak absolute values of the responses, each based on its static value of Fig. 1. These are the highest peaks which occurred among all the spans traversed (from 5 to 20), and occurred in the range $0.5 \leqq (x/s) \leqq 0.65$. Above frequency ratios of about 0.6, these peaks occurred during free span vibration. The experimental points in Fig. 2 are for a moving point mass where $M_T/\rho As = 1/2$ as well.[4] Agreement is good.

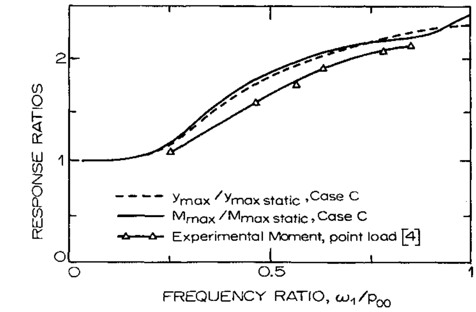


Fig. 2 Peak span responses for a mass ratio of one-half.

Table 1 shows responses for the other cases. For case A, $M_T = M$ and for cases B to E, $M_T = 2M$. The frequency ratios cover the peaks of the response curves and also lie within the range of practical design. For instance, for a 75-ft span of frequency $p_{00} = 15.0$ rad/sec and for a vehicle speed $v = 300$ mph, the passage frequency is $\omega_1 = 18.3$ and $\omega_1/p_{00} = 1.22$. Typically, $M_T/\rho As \simeq 1/2$. If $d/s = 1/3$ and two masses travel together (case C) the dynamic deflection and moment exceed their static values by 2.27 and 2.66, respectively.

Results show that as the total length of the moving mass approaches the span length, responses are bounded; but as the mass lengths exceed the span length (case E) responses grow and may become unbounded. An increase in mass ratio causes higher responses, other parameters remaining the same. Finally, a 5% damping ratio reduces responses from 5%–10%.

References

[1] Wilson, J. F., "Response of the Simple Spans to Moving Mass Loads," AIAA Paper 72-338, San Antonio, Texas, 1972.

[2] Wilson, J. F., "Dynamic Response of Beams to Moving Pressure Loads as Related to Tracked Air Cushion Vehicles," *The Shock and Vibration Bulletin*, Vol. 40, No. 4, Dec. 1969, pp. 47–59.

[3] Nelson, H. D. and Conover, R. A., "Dynamic Stability of a Beam Carrying Moving Masses," ASME Preprint 71-APM-M, 1971.

[4] Ayre, R. S., Jacobson, L. S., and Hsu, C. S., "Transverse Vibration of One and of Two-Span Beams Under the Action of a Moving Mass Load," *Proceedings of the First U.S. National Congress of Applied Mechanics*, 1951, pp. 81–90.